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**Trajectory Optimization for Magnetorquer-Based Underactuated Control of Small Satellites**

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**ABSTRACT**

Due to the ever-increasing scope of small satellite missions, there is now significant demand for precise attitude determination and control capabilities onboard CubeSats. Interactions between magnetic torque coils and the Earth’s magnetic field have been used for decades onboard satellites to offload momentum from reaction wheels and control-moment gyro- scopes. However, magnetorquers are inherently underactuated, and mechanical actuators like reaction wheels are often prohibitively expensive in terms of mass, volume, power, and cost for CubeSat missions.

This paper presents a magnetorquer-only attitude control technique that utilizes trajectory optimization to circumvent the under-actuated nature of satellite magnetic field interactions. Given a known or- bit and desired attitude state, the method utilizes a simplified dynamics model and a fast constrained trajectory optimization solver based on differential dynamic programming to arrive at a nominal torque profile that respects the spacecraft’s actuator limitations and desired actuation efficiency. This nominal maneuver is then tracked online using a time-varying linear-quadratic regulator (LQR). Using realistic parameters for a 1U CubeSat in low-Earth orbit, this scheme is able to perform an arbitrary 90◦ reorientation within a few minutes.

To demonstrate the effectiveness and robustness of the proposed control technique, we present the results of several high-fidelity closed-loop simulations using the IGRF magnetic field model, NRLMSISE- 00 atmosphere model, and solar radiation pressure effects. We also discuss the computational complexity of the method and future implementation in a CubeSat flight computer.

**INTRODUCTION**

In this section, I will talk about briefly the history of what magnetorquers have been used for on satellites, as well as discussing their continuing and larger use on small satellites in the place of more traditional kinds of attitude control devices like CMGs and reaction wheels. Probably will also cite some recent papers that suggest using magnetoruqers for more control in small satellites.

**CONCEPT VALIDATION**

In this section, I will go through three different things that will help prove the case for further investigation into magnetorquer only control.

***Satellite Scaling***

In this section, I will discuss how the magnetorquer magnetic moment increases as a squared term with satellite size, since it is dependent on the area of the face on which it is acting. On the other hand, for a box with constant density, the moment of inertia goes up on the order of the 5th against the satellite size. Therefore, using magnetorquers for control becomes more and more useful with smaller and smaller satellites.

*Equation on torque from magnetorquer*

*Equation on moment of inertia for satellite of constant density*

***Efficiency and Jitter***

In this section, I will discuss how the magnetorquer uses less electric power per unit torque to actuate the 1U satellites compared to the Blue Canyon Tech FleXcore momentum wheels, and it is therefore more efficient. Also, I should touch on the fact that magnetorquers have inherently no jitter, provided that the control input is not subject to a high frequency and complex impedance does not have a large part.

*Equation on power required for a certain amount of angular work comparing magnetorquers to FleXcore*

***Magnetic Field***

In this section, I will talk about the IGRF magnetic field over the surface of the Earth, and make the argument that it is strong enough all over the globe to be able to provide control authority to a reasonable magnetorquer-only control system independent of the orbit elements. I will also comment on the decreasing magnetic field compared to the height of the spacecraft, especially to emphasize that this system works primarily for small satellites in LEO.

*Image of the magnetic field strength*

***Careful COTS***

I want to briefly touch on the improvements made to the computational power to small satellites that allow such significant control inputs to be found in real time on orbit rather than requiring a larger computer on the ground to uplink the commands. Also, the satellite control system will be stronger.

**SIMULATION**

I will give details about the setup of the simulation used to calculate the satellite slew maneuver and attitude control.

***Orbit Propagation and Magnetic Field Measurement***

Given some initial orbital parameters, the orbit was propagated using a traditional orbit propagator that includes oblateness (J2) and atmospheric drag. Once the ECI position has been calculated over the chosen time segment, it is translated to an ECEF coordinate system and then to latitude and longitude in a geodetic approximation, and the magnetic field over that time segment is found using an IGRF lookup table.

*Equation needed?*

***Trajectory Optimization***

The initial control input was calculated using a constrained optimization solver employing iLQR and Augmented Lagrangians, both of which I should discuss in cursory detail. The magnetic field at each knot point is found by looking up the appropriate corresponding normal magnetic field that was previously found and then translating it to the body frame of the spacecraft given the quaternion.

*Equation for cost function*

*Equation for linearized dynamics*

*Equation for delta control input (from iLQR paper)*

***Time-Varying LQR Simulation***

Once the control input is calculated by the trajectory optimization solver, the control input is simulated in a feedback loop that utilizes a time-varying LQR formulation. I should explain how the TVLQR control system works, especially how the gain matrices are calculated and such. Then I should probably go into some more detail on the differences between the function used to find the optimal trajectory and the simulation function. This will change depending on the simulations that I’m running (for instance, the variance of the gaussian noise implemented to simulate for sensor noise).

*Equation for cost to go*

*Equations for simulation time step*

**RESULTS**

In this section, I explain the different types of cases that I ran the simulation for and what motivates these cases as examples that I’d want to measure.

***CubeSat Properties***

I should probably talk a little bit about the properties of the CubeSats that I use in these simulations. I’m going to talk about the 1U (mass, MOI, etc.) and the 3U, and then get into the sensors that they employ for attitude, gyro, and magnetometer detection. I’ll probably also cite the nominal noise that these sensors should experience in this section.

*Table with properties of 1U*

*Table with properties of 3U*

*Table with nominal sensor noise*

***1U Pointing***

In this case, I want to measure the corner cases on slewing a 1U cubesat 90 degrees and attempt to decipher how fast I can get the thing to turn at different orbits and points in orbits.

*Image of state convergence and control input*

***1U Attitude Control***

In addition to the slew maneuvering, I want to measure the ability of the system to keep the system pointed at a specific orientation (like for telescopes or Earth observation). Then I’ll start cranking up the noise until the thing breaks, and report when that happens.

*Image of control input and state (should I have the error too?)*

***3U Pointing***

Again, want to measure the alacrity of the control system to turn a 3U CubeSat 90 degrees or so.

*Image of state convergence and control input*

***3U Nadir Attitude Control***

For this case, I want to spec out a 3U telescope satellite and see if I can get it to stay trained within 1 degree on a point on Earth over a large swath of a pass to try to validate this for application to a real business case.

*Image of state and control input*

**CONCLUSION**

In the conclusion, I just want to recap on the reasons why this is valid (the satellite size scaling argument, the efficiency of the magnetorquers versus conventional attitude control methods, the increased computational power of small satellites through careful COTS, and lack of jitter). A terse summary of the simulation would be useful (just to go back through the satellite position calculation, the determination of an optimal trajectory, and then the simulation using TVLQR control system. Then I’ll go through the cases that the simulation enabled and talk about their use cases. For the 1U, it facilitates the use of 1U CubeSats for telescope missions that otherwise would not have been possible. For the 3U, it would enable a powerful alternative to the attitude control with reaction wheels currently employed, and also facilitate a jitter-free nadir pointing algorithm for telescope missions that require fast slewing.

***Acknowledgments***

Yo Taylor, I'm really happy for you, Imma let you finish but Beyonce had one of the best videos of all time. Long live Queen B.

***Notes***

***References***